NTNU | Norwegian University of Science and Technology

HASHES AND MACS

TTM4135 - Lecture 6

Tjerand Silde

22.01.2026

Motivation

- Hash functions are versatile cryptographic functions used as a building block for authentication
- Message authentication codes (MACs) are a symmetric key cryptographic mechanisms providing authentication and integrity services
- The standardized MAC, HMAC, can be based on many different hash functions and is often used in the TLS protocol
- Authenticated encryption combines confidentiality and authentication in one mechanism, combining encryption and a MAC
- GCM is a standardized authenticated encryption algorithm used in TLS

Outline

Contents

Hash functions

A *hash function* H is a public function such that:

- H is simple and fast to compute
- H takes as input a message M of arbitrary length and outputs a message digest $H(M)$ of fixed length

Security properties of hash functions

Collision resistant:

- It should be infeasible to find values $x_1 \neq x_2$ with $H(x_1) = H(x_2)$.

One-way (or preimage resistant):

- Given a value y it should be infeasible to find any x with $H(x) = y$.

Second-preimage resistant:

- Given a value x_1 it should be infeasible to find $x_1 \neq x_2$ with $H(x_1) = H(x_2)$.

An attacker who can break second-preimage resistance can also break collision resistance, so the latter is a stronger security property.

The birthday paradox

- In a group of 23 randomly chosen people, the probability that at least two have the same birthday is over 0.5.
- In general, if we randomly choose around $\sqrt{M}$ values from a set of size M, the probability of getting two equal values is around 0.5.
- If a hash function H has an output size of k bits, then $2^{k/2}$ trials are enough to find a collision of two elements with probability around 0.5.
- 2^{128} trials are considered infeasible, therefore, in order to satisfy collision resistance, hash functions should have output of at least 256 bits.

Birthday paradox example, $M = 100$

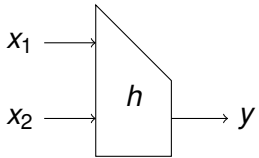
# trials	Collision prob.	# trials	Collision prob.
1	0	13	.55727
2	.01000	14	.61483
3	.02980	15	.66876
4	.05891	16	.71845
5	.09656	17	.76350
6	.14174	18	.80371
7	.19324	19	.83905
8	.24972	20	.86964
9	.30975	21	.89572
10	.37188	22	.91762
11	.43470	23	.93575
12	.49689	24	.95053

Iterated hash functions

- Cryptographic hash functions need to take arbitrary-sized input and produce a fixed size output.
- As we saw from block ciphers, we can process arbitrary-sized data by having a function that processes fixed-sized data and use it repeatedly.
- An *iterated hash function* splits the input into blocks of fixed size and operates on each block sequentially using the same fixed size function.
- Merkle–Damgård construction: use a fixed-size *compression function* applied to multiple blocks of the message.

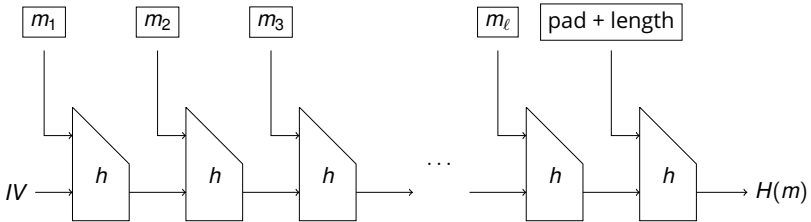
Compression function *h*

h takes two *n*-bit input strings x_1 and x_2 and produces an *n* bit output string *y*.



Merkle–Damgård construction

1. Break message m into n-bit blocks $m_1 \| m_2 \| \ldots \| m_\ell$.
2. Add padding and an encoding of the length of m.
   This process may, or may not, add one block.
3. Input each block into compression function h along with chained output; use IV to get started.



Use of Merkle–Damgård construction

Security: If compression function *h* is collision-resistant,
then hash function *H* is collision-resistant.

But also some security weaknesses:

- Length extension attack: once you have one collision, easy to find more
- Second-preimage attacks not as hard as they should be

Many standard, and former standard, hash functions are
Merkle–Damgård constructions: MD5, SHA-1, SHA-2 family.

MDx family

- Proposed by Rivest and widely used throughout 1990s
- Deployed family members were MD2, MD4 and MD5
- All have 128 bit output strings
- All of them are broken (concrete collisions have been found)
- In 2006, MD5 collisions could be found in one minute on a PC

SHA-0 and SHA-1

- Based on MDx family design but larger output and more complex
- Originally Secure Hash Algorithm published by US standard agency NBS (now called NIST) in 1993 and later given name SHA-0
- Replaced by SHA-1 in 1995 with very small change to algorithm
- Both SHA-0 and SHA-1 have 160 bit output strings
- SHA-0 has been broken (collisions found in 2004)
- First SHA-1 collision found in 2017 - attack is 100 000 times faster than brute force search

SHA-2 family

Developed in response to (real or theoretical) attacks on MD5 and SHA-1.

	Hash size	Block size	Security match
SHA-224	224 bits	512 bits	2 key 3DES
SHA-512/224	224 bits	1024 bits	2 key 3DES
SHA-256	256 bits	512 bits	AES-128
SHA-512/256	256 bits	1024 bits	AES-128
SHA-384	384 bits	1024 bits	AES-192
SHA-512	512 bits	1024 bits	AES-256

- Standardized in FIPS PUB 180-4 (August 2015)
- Known collectively as SHA-2

Padding in the SHA-2 family

- The message length field is:
  - 64 bits when the block length is 512 bits
  - 128 bits when the block length is 1024 bits
- There is always at least one bit of padding[1]. After the first '1' in the padding, enough '0' bits are added so that after the length field is added there is an exact number of complete blocks.
- Adding the padding and length field sometimes adds an extra block and sometimes does not.

[1] To avoid trivial second preimage attacks.

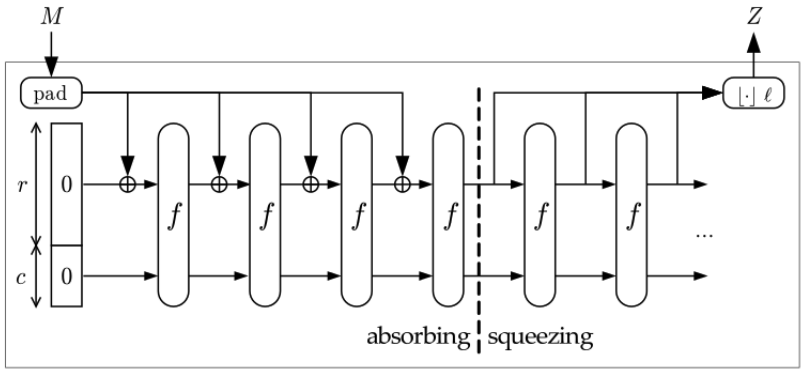
SHA-3

- Late 2000s seen to be a crisis in hash function design
- MDx and SHA family are all based on the same basic design and there have been several unexpected attacks on these in recent years
- In November 2007, NIST announced a competition for a new hash standard called SHA-3
  - Entries closed October 2008; 64 original candidates
  - 14 went through to Round 2, with 5 finalists announced in December 2010
  - Keccak selected as winner in October 2012.
  - Keccak does not use compression function as in Merkle–Damgård construction. Instead it uses a *sponge* construction.
  - Standardized in FIPS PUB 202, August 2015

The sponge construction

- Input is padded and broken down into blocks of r bits
- The b bits of the state are initialized to zero, and the sponge function proceeds in the following way:
  - Absorbing phase: the input blocks are XOR'ed into the r first bits of the state, and the function f is applied iteratively
  - Squeezing phase: the first r bits of the state are returned as output blocks, interleaved with applying the function f.
    - The number of output blocks is chosen by the user.
    - The last c bits of the state are never directly affected by the input blocks and are never output during the squeezing phase.
  - Since the input-output sizes can be arbitrarily long, the sponge construction can be used to build many primitives (hash functions, stream ciphers, MAC).
    - Long input, short output $\rightarrow$ Hash function
    - Short input, large output $\rightarrow$ Key stream

The Keccak function



Source: https://keccak.team/sponge_duplex.html

Uses of hash functions

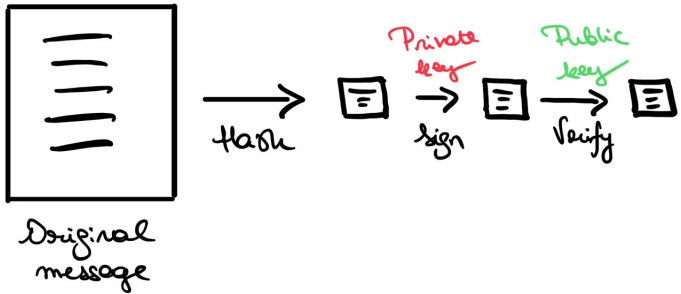
Hash functions have many uses, but applying a hash is *not* encryption:

- Hash computation does not depend on a key.
- It is not designed to go backwards to find the input.

While hash functions alone do not provide data authentication, they often help in achieving it:

- Authenticate the hash of a message to authenticate the message.
- Building block for Message Authentication Codes (see HMAC below).
- Building block for signatures (later lecture).

Reminder: signatures



Hash functions and keys

- Sometimes we write hash functions as taking a key s as $H^s(x) = H(s, x)$
- It must be hard to find a collision for a randomly generated key s
- The key is *not* kept secret; collision resistance must hold even if the adversary has the key s

Storing passwords for login

Common to store salted hashes of user passwords on servers: pick random string *salt*, compute $h = H(pw, salt)$, store $(salt, h)$:

- easy to check password pw': compare stored h and computed $H(pw', salt)$
- hard to recover pw from $h = H(pw)$ assuming H is preimage resistant
- attacker needs to store a different dictionary for each *salt*

Note that using a *slower* hash function slows down password guessing.

Contents

Message Authentication Codes (MAC)s

- Recall one of the goals of cryptography is to enable *secure communications*
  - But what does this mean?
- We have covered *secrecy* so far (i.e. *hiding* the message)
- But *integrity can be even more important*
- For example, a bank receives a transfer request from user Alice to user Bob in the amount of $ 10,000
  - Did it really come from Alice? Did someone else send it?
  - Is the amount correct? Was it modified during transmission?

Encryption vs Message Authentication

- Encryption *does not* guarantee message integrity
- These are *different* security notions
- Recall that we saw that flipping certain bits in the ciphertext results in flipping certain bits in the plaintext
- If the adversary also has partial information about the plaintext (e.g. an estimate of the amount that is being sent), it can predict with some accuracy what the changes are
  - Even the OTP is vulnerable to this, so this does not mean that the encryption scheme is not secure
- An adversary can also randomly change the ciphertext, to ruin the underlying message

Message Authentication Code (MAC)s

- A *message authentication code (MAC)* is a cryptographic mechanism used for message integrity and authentication
- On input a secret key K and an arbitrary length message M, a MAC algorithm outputs a fixed-length tag:

$$T = \mathrm{MAC}(M, K)$$

- A MAC is a symmetric key algorithm: sender and receiver both have the secret key K
- The sender sends the pair (M, T) but M may or may not be encrypted
- The recipient recomputes the tag $T' = \mathrm{MAC}(M', K)$ on the received message M' and checks that $T' = T$

MAC properties

The basic security property of a MAC is called *unforgeability*:

- It is not feasible to produce a message M and a tag T such that $T = \mathrm{MAC}(M, K)$ without knowledge of the key K

The complete notion of security is *unforgeability under chosen message attack*:

- The attacker is given access to a *forging oracle*: on input any message M of the attacker's choice the MAC tag $T = \mathrm{MAC}(M, K)$ is returned
- It is not feasible for the attacker to produce a valid (M, T) pair that was not already asked to the oracle

A MAC is *not* a signature scheme, but can be thought of as the symmetric version of a signature. Only *authorized* entities can authenticate messages.

HMAC

- Proposed by Bellare, Canetti, Krawczyk in 1996
- Built from any iterated cryptographic hash function *H*, e.g. SHA-256
- Standardized and used in many applications including TLS and IPsec
- Details in FIPS-PUB 198-1 (July 2008)

HMAC construction

Let H be any iterated cryptographic hash function. Then define:

$$\mathrm{HMAC}(M, K) = H(\,(K \oplus \mathrm{opad}) \,\|\, H((K \oplus \mathrm{ipad}) \,\|\, M)\,)$$

- M: message to be authenticated
- K: key padded with zeros to be the block size of H
- opad: fixed string `0x5c5c5c...5c`
- ipad: fixed string `0x363636...36`
- $\|$ denotes concatenation of bit strings.

HMAC is secure (unforgeable) if H is collision resistant or if H is a pseudorandom function.

HMAC

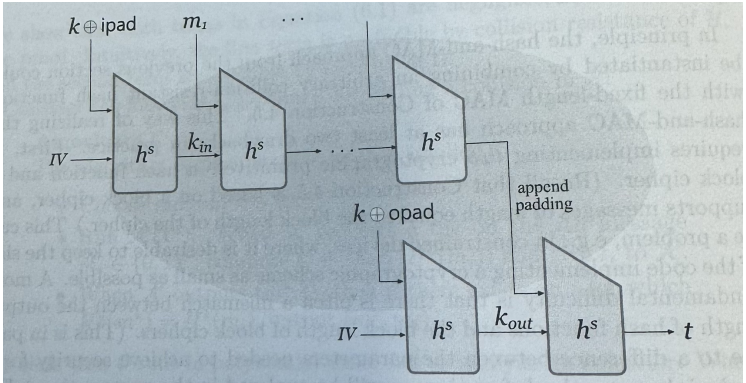


FIGURE 6.2: HMAC, pictorially.

Source: Katz-Lindell book, third edition.

Security of HMAC

- *Security:* HMAC is secure if H is collision resistant or if H is a pseudorandom function.
- HMAC is designed to resist length extension attacks (even if H is a Merkle–Damgård hash function).
- HMAC is often used as a *pseudorandom function* for deriving keys in cryptographic protocols.

H is h^s in the previous slide.

Contents

Authenticated encryption

Suppose Alice and Bob have a shared key K.

Suppose Alice has a message M that she wants to send to Bob with *confidentiality* and *authenticity/integrity*.

How should she do this? Two options:

1. Split the key K into two parts (K_1 and K_2), encrypt with K_1 to provide confidentiality and use K_2 with a MAC to provide authenticity and integrity.
2. Use a dedicated algorithm which provides both properties – this is called *authenticated encryption*.

Combining encryption and message authentication

Three possible ways to combine encryption and MAC are:

Encrypt-and-MAC: encrypt M, apply MAC to M, and send the two results

MAC-then-encrypt: apply MAC to M to get tag T, then encrypt $M \| T$, and send the ciphertext

Encrypt-then-MAC: encrypt M to get ciphertext C, then MAC C, and send the two results

It turns out that *encrypt-then-MAC* is the safest approach:

- $C = \mathrm{Enc}(M, K_1)$
- $T = \mathrm{MAC}(C, K_2)$
- send $C \| T$

Encrypt-and-MAC

- No integrity on the ciphertext! Only on the plaintext, which is problematic.
- Even a strongly secure MAC does not guarantee *anything* about secrecy.
  - Think of a MAC who always outputs the first bits of m in the tag.

MAC-then-Encrypt

- Plaintext integrity only, but this time the MAC is encrypted.
- Because we pad the message with the tag, we have two sources of decryption error:
  - Padding may be incorrect.
  - Tag may not verify.
- An attacker can distinguish between the failures and exploit this.
- This type of attack has been carried out against TLS configurations.

Authenticated encryption with associated data (AEAD)

- An AEAD algorithm is a symmetric key cryptosystem
- Inputs to the AEAD encryption process are:
  - a message *M*
  - associated data *A*
  - the shared key *K*
- The AEAD output *O* may contain a ciphertext and tag
- The sender sends *O* and *A* to the recipient
- The receiver either accepts the message *M* and data *A*, or reports *failure*
- Any AEAD algorithm should provide
  - confidentiality for *M*
  - authentication for both *M* and *A*

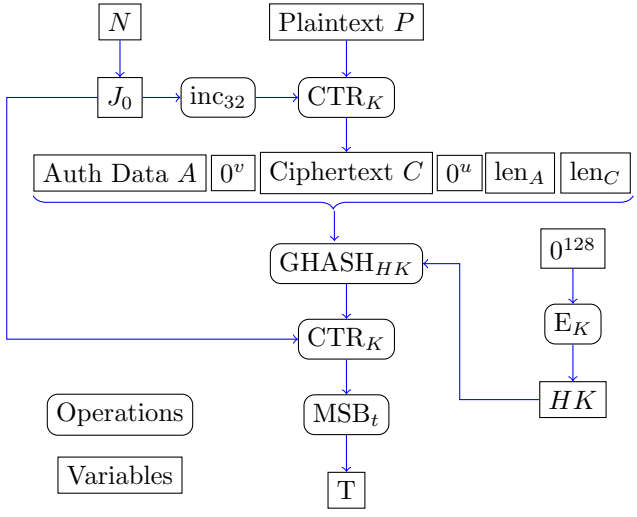
Galois Counter Mode (GCM)

- A block cipher mode providing AEAD
- Most commonly used mode in web-based TLS
- Combines CTR mode on a block cipher (typically AES) with a special keyed hash function called GHASH.
- Standard definition in NIST SP-800 38D
- Due to hardware support of AES and carry-less addition in modern Intel chips, GCM using AES can be faster than using AES with HMAC.

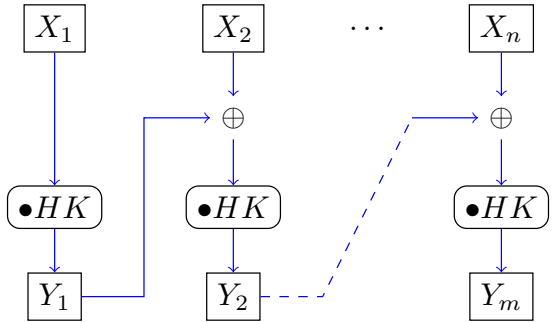
GCM algorithm

- GHASH uses multiplication in the finite field $GF(2^{128})$
  - Generated by the polynomial $x^{128} + x^7 + x^2 + 1$.
- Inputs are plaintext P, authenticated data A, and nonce N
- Values u and v are the minimum number of 0s required to expand A and C to complete blocks
- Outputs are ciphertext C and tag T. The length of A, len_A, and the length of C, len_C, are 64-bit values
- In TLS the length of T is $t = 128$ bits and the nonce N is 96 bits. The initial block input to CTR mode of E (denoted CTR in diagram) is $J_0 = N \parallel 0^{31} \parallel 1$.
- The function inc_{32} increments the right-most 32 bits of input by 1 mod 2^{32}

GCM algorithm



GHASH



- Output is $Y_m = GHASH_{HK}(X_1, \ldots, X_m)$
- Operation $\bullet$ is multiplication in the finite field $GF(2^{128})$
- $HK = E(0^{128}, K)$ is the hash subkey.

GCM decryption

- The elements transmitted to the receiver are the ciphertext C, the nonce N, the tag T and the authenticated data A.
- All elements required to recompute the tag T are available to the receiver who shares key K. The tag is recomputed and checked with received tag. If tags do not match then output is declared invalid.
- If the tag is correct then the plaintext can be recomputed by generating the same key stream, from CTR mode, as is used for encryption.

Contents

Cryptography and passwords

Cryptography needs high-entropy secrets:

- RC4, AES-128: 128 bit secret key
- HMAC-SHA1: 160-bit secret key
- AES-256, HMAC-SHA256: 256-bit secret key

128 bits = about 23 character alphanumeric secret

Humans can only memorize low entropy passwords:

- RockYou.com password database compromised in 2009; *password entropy 21.1 bits*

Uses of passwords

Applications of passwords:

- *Login*: system stores password to compare with the value the user types at login to decide whether to allow access. Obviously do not want to store passwords in plaintext on disk.

- *Key derivation*: user remembers a password that will be used to derive a key for encryption, e.g., disk encryption.

Dictionary attacks against passwords

- Attacker obtains a dictionary of passwords sorted by approximate frequency of use.
- Attacker iterates through dictionary from most frequent to least frequent passwords.

Storing passwords for login

How can we store user passwords on hard disks for checking at login?

- Store passwords in plaintext: Bad idea; anyone who gets hard disk can learn password.
- Store passwords encrypted using a secret key: Where do you store the secret key? Becomes a chicken-and-egg problem.
- Store hashes of passwords: $h = H(pw)$
  - Pro: easy to check password pw': compare stored h and computed $H(pw')$
  - Pro: hard to recover pw from $h = H(pw)$ assuming H is preimage resistant
  - Con: attacker could store a dictionary of $pw, H(pw)$ and compare with h

Storing passwords for login

How can we store user passwords on hard disks for checking at login?

- Salted hashes: pick random *salt*, compute $h = H(pw, salt)$, store $(salt, h)$
  - Pro: easy to check password pw': compare h and computed $H(pw', salt)$
  - Pro: hard to recover pw from $h = H(pw)$ assuming H is preimage resistant
  - Pro: attacker needs to store a different dictionary for each *salt*
  - Con: does not slow down per-password attacks

PBKDF2

PBKDF2 = Password-Based Key Derivation Function 2

- uses a hash function, MAC, or cipher
- combines password and salt
- uses *multiple iterations* to slow down attacks
- can output a key of arbitrary length

Used in:

- login password storage: Apple iOS, Mac OS X
- network login: WPA and WPA2
- disk encryption: TrueCrypt, Mac OS X FileVault, Android, …

PBKDF2 construction

Let F be a pseudorandom function, such as HMAC with hash function SHA256. Suppose we want 256 bits of output key.

$$\text{PBKDF2}(P, pw, salt, c) = U_1 \oplus U_2 \oplus U_3 \oplus \cdots \oplus U_c$$

where

- $U_1 = F(pw, salt \| 1)$
- $U_j = F(pw, U_{j-1})$ for $j \geq 2$

Originally $c = 1000$ iterations were recommended, but modern applications may want more (e.g., iOS 4 has $c = 10000$).

Questions?